January, 2003

Hadronic Charmed Meson Decays Involving Axial Vector Mesons

Hai-Yang Cheng

Institute of Physics, Academia Sinica
Taipei, Taiwan 115, Republic of China

Abstract

Cabibbo-allowed charmed meson decays into a pseudoscalar meson and an axial-vector meson are studied. The charm to axial-vector meson transition form factors are evaluated in the Isgur-Scora-Grinstein-Wise quark model. The dipole momentum dependence of the $D \to K$ transition form factor and the presence of a sizable long-distance W-exchange are the two key ingredients for understanding the data of $D \to \overline{K} a_1$. The $K_{1A} - K_{1B}$ mixing angle of the strange axial-vector mesons is found to be $\approx \pm 37^\circ$ or $\pm 58^\circ$ from $\tau \to K_1 \nu_\tau$ decays. The study of $D \to K_1(1270)\pi,\ K_1(1400)\pi$ decays excludes the positive mixing-angle solutions. It is pointed out that an observation of the decay $D^0 \to K_1^-(1400)\pi^+$ at the level of 5×10^{-4} will rule out $\theta \approx -37^\circ$ and favor the solution $\theta \approx -58^\circ$. Though the decays $D^0 \to \overline{K}_1^0 \pi^0$ are color suppressed, they are comparable to and even larger than the color-allowed counterparts: $\overline{K}_1^0(1270)\pi^0 \sim K_1^-(1270)\pi^+$ and $\overline{K}_1^0(1400)\pi^0 > K_1^-(1400)\pi^+$. The finite width effect of the axial-vector resonance is examined. It becomes important for $a_1(1260)$ in particular when its width is near 600 MeV.

I. INTRODUCTION

Two-body hadronic D decays containing an axial-vector meson in the final state have been studied in [1–6]. There are two different types of axial vector mesons: ${}^3P_1$ and ${}^1P_1$, which carry the quantum numbers $J^{\rm PC} = 1^{++}$ and 1^{+-}, respectively. The non-strange axial vector mesons, for example, $a_1(1260)$ and $b_1(1235)$ which correspond to ${}^3P_1$ and ${}^1P_1$, respectively, cannot have mixing because of the opposite C-parities. On the contrary, the strange partners of $a_1(1260)$ and $b_1(1235)$, namely, K_{1A} and K_{1B}, respectively, are not mass eigenstates and they are mixed together due to the strange and non-strange light quark mass difference.

It has been noticed for a long time that the predicted $D^0 \to K^- a_1^+$ and $D^0 \to \overline{K}^0 a_1^+$ rates are too small by a factor of 5-6 and 2, respectively, when compared with experiment [1–5]. Interestingly, the predicted $D^0 \to K_1^-(1270)\pi^+$ and $D^+ \to \overline{K}_1^0(1400)\pi^+$ are also too small by roughly a factor of 5 and 2, respectively, compared to the data [5]. One argument is that the factorization approach may be only suitable for energetic two-body decays; for $D \to \overline{K} a_1(1260)$ with very little energy release, the approximation is questionable [2]. Since $a_1(1260)$ is a broad resonance which will increase the phase space available, it is thus expected that the threshold suppression can be obviated. However, a detailed study of the a_1 mass smearing effect does not show the desired enhancement [1,5]. Therefore, $D^0 \to K^- a_1^+$ and $D^0 \to \overline{K}^0 a_1^+$ remain a problem. Compared to the ρ production, we see experimentally that $\mathcal{B}(D^+ \to \overline{K}^0 a_1^+) \gtrsim \mathcal{B}(D^+ \to \overline{K}^0 \rho^+)$ and $\mathcal{B}(D^0 \to K^- a_1^+) \lesssim \mathcal{B}(D^0 \to K^- \rho^+)$ [7]. Although the phase space for $\overline{K} a_1(1260)$ is largely suppressed relative to that for $\overline{K}\rho$, the large $a_1(1260)$ production comparable to ρ is quite interesting. It is important to understand these features.

The purpose of this work is to reexamine the axial-vector meson production in the charmed meson decays and to resolve the aforementioned long-standing problems.

The study of charm decays into an axial-vector meson and a pseudoscalar meson will require the knowledge of form factors and decay constants. In the early study of [5], the charm to axial vector meson transition form factors are calculated using the ISGW (Isgur-Scora-Grinstein-Wise) model [8]. However, some of the form factors get substantial modifications in the improved version of the ISGW model, the so-called ISGW2 model [9]. For example, the relevant $D \to a_1(1260)$ and $D \to K_{1A}$ transition form factors can be different by a factor of 3 in the ISGW and ISGW2 models. In the present paper we will use the ISGW2 model to compute the charm to axial-vector meson transition form factors, and we find that $D \to \overline{K} a_1(1260)$ decays provide a nice probe of the momentum dependence of the $D \to K$ transition form factor at large q^2.

It is known from the data analysis based on the model-independent diagrammatic approach [10,11] that weak annihilation (W-exchange or W-annihilation) in charm decays is quite sizable as it can receive large contributions from final-state interactions via quark rescattering. We shall show that the W-exchange contribution is one of the key ingredients for understanding the data.

The paper is organized as follows. In Sec. II we will discuss the decay constants and form factors relevant for our purposes. The $D \to AP$ decays are then discussed in detail in Sec. III. Sec. IV gives our conclusions. An Appendix is devoted to a sketch of the ISGW model for the $D \to A$ transition form factor calculations.

II. DECAY CONSTANTS AND FORM FACTORS

In the present work we consider the isovector non-strange axial vector mesons $a_1(1260)$ and $b_1(1235)$ and the isodoublet strange ones $K_1(1270)$ and $K_1(1400)$. Their masses and widths are summarized in Table I. The axial vector mesons $a_1(1260)$ and $b_1(1235)$ have the quantum numbers $^3P_1$ and $^1P_1$, respectively. They cannot have mixing because of the opposite C-parities. However, $K_1(1270)$ and $K_1(1400)$ are a mixture of $^3P_1$ and $^1P_1$ states owing to the mass difference of the strange and non-strange light quarks. We write

$$\begin{aligned} K_1(1270) &= K_{1A}\sin\theta + K_{1B}\cos\theta, \\ K_1(1400) &= K_{1A}\cos\theta - K_{1B}\sin\theta, \end{aligned} \tag{2.1}$$

where K_{1A} and K_{1B} are the strange partners of $a_1(1260)$ and $b_1(1235)$, respectively. If the mixing angle is 45° and $\langle K\rho|K_{1B}\rangle = \langle K\rho|K_{1A}\rangle$, one can show that $K_1(1270)$ is allowed to decay into $K\rho$ but not $K^*\pi$, and vice versa for $K_1(1400)$ [12].

TABLE I. The masses and widths of the $1\,^3P_1$ and $1\,^1P_1$ axial-vector mesons quoted in [7].

	$a_1(1260)$	$b_1(1235)$	$K_1(1270)$	$K_1(1400)$
mass	1230 ± 40 MeV	1229.5 ± 3.2 MeV	1273 ± 7 MeV	1402 ± 7 MeV
width	$250 - 600$ MeV	142 ± 9 MeV	90 ± 20 MeV	174 ± 13 MeV

From the experimental information on masses and the partial rates of $K_1(1270)$ and $K_1(1400)$, Suzuki found two possible solutions with a two-fold ambiguity, $|\theta| \approx 33°$ and 57° [13]. A similar constraint $35° \lesssim |\theta| \lesssim 55°$ is obtained in [14] based solely on two parameters: the mass difference of the a_1 and b_1 mesons and the ratio of the constituent quark masses.

Based on the early data from the TPC/Two-Gamma Collaboration [15]

$$\begin{aligned} \mathcal{B}(\tau^- \to K_1^-(1270)\nu_\tau) &= (4.1^{+4.1}_{-3.5} \pm 1.0) \times 10^{-3}, \\ \mathcal{B}(\tau^- \to K_1^-(1400)\nu_\tau) &= (7.6^{+4.0}_{-3.3} \pm 2.0) \times 10^{-3}, \end{aligned} \tag{2.2}$$

Suzuki has shown that the observed $K_1(1400)$ production dominance in the τ decay favors $|\theta| \approx 33°$ [13]. However, the analysis by ALEPH Collaboration based on the LEP data yields [16]

$$\begin{aligned} \mathcal{B}(\tau^- \to K_1^-(1270)\nu_\tau) &= (4.8 \pm 1.1) \times 10^{-3}, \\ \mathcal{B}(\tau^- \to K_1^-(1400)\nu_\tau) &= (0.5 \pm 1.7) \times 10^{-3}. \end{aligned} \tag{2.3}$$

This indicates that $K_1(1400)$ production is somewhat reduced in comparison with that of $K_1(1270)$. Assuming the resonance structure of $\tau^- \to K^-\pi^+\pi^-\nu_\tau$ decays being dominated by $K_1^-(1270)$ and $K_1^-(1400)$, both OPAL [17] and CLEO [18] have also measured the ratio of $K_1(1270)\nu_\tau$ to $K_1(1400)\nu_\tau$ with the averaged result [7]

$$\frac{\Gamma(\tau \to K_1(1270)\nu_\tau)}{\Gamma(\tau \to K_1(1270)\nu_\tau) + \Gamma(\tau \to K_1(1400)\nu_\tau)} = 0.69 \pm 0.15\,. \tag{2.4}$$

This in turn implies that

$$R \equiv \frac{\mathcal{B}(\tau \to K_1(1270)\nu_\tau)}{\mathcal{B}(\tau \to K_1(1400)\nu_\tau)} = 2.2 \pm 1.2\,. \tag{2.5}$$

Therefore, the new data clearly show $K_1(1270)$ dominance in the τ decay. Consequently, the previous argument of ruling out $|\theta| \approx 57^\circ$ from $K_1(1400)$ production dominance is thus no longer valid. This will be elaborated in more detail shortly below.

A. Decay constants

The decay constant of the axial-vector meson is defined by

$$\langle 0|A_\mu|A(q,\varepsilon)\rangle = f_A m_A \varepsilon_\mu. \tag{2.6}$$

Because of charge conjunction invariance, the decay constant of the $^1P_1$ non-strange neutral meson $b_1(1235)$ must be zero. In the isospin limit, the decay constant of the charged b_1 must vanish, so that f_{b_1} is small. As for the strange axial vector mesons, the $^3P_1$ and $^1P_1$ states transfer under charge conjunction as

$$M_a^b(^3P_1) \to M_b^a(^3P_1), \qquad M_a^b(^1P_1) \to -M_b^a(^1P_1), \quad (a,b=1,2,3). \tag{2.7}$$

Since the weak axial-vector current transfers as $(A_\mu)_a^b \to (A_\mu)_b^a$ under charge conjunction, it is clear that $f_{K_{1B}} = 0$ in the SU(3) limit [13].

For $a_1(1260)$ and K_{1A}, their decay constants can in principle be determined from the τ decay. From the measured $\tau \to K_1^-(1270)\nu_\tau$ from ALEPH, the decay constant of $K_1(1270)$ is extracted to be

$$f_{K_1(1270)} = 175 \pm 19 \text{ MeV}, \tag{2.8}$$

where use has been made of the formula

$$\Gamma(\tau \to K_1\nu_\tau) = \frac{G_F^2}{16\pi}|V_{us}|^2\, f_{K_1}^2 \frac{(m_\tau^2 + 2m_{K_1}^2)(m_\tau^2 - m_{K_1}^2)^2}{m_\tau^3}. \tag{2.9}$$

To determine the decay constant of $K_1(1400)$ we note that $f_{K_1(1400)}/f_{K_1(1270)} = \cot\theta$ in the exact SU(3) limit. However, the decay constant of K_{1B} is non-zero beyond the SU(3) limit. We thus follow [13] to write

$$\frac{m_{K_1(1400)} f_{K_1(1400)}}{m_{K_1(1270)} f_{K_1(1270)}} = \frac{\cos\theta + \delta \sin\theta}{\sin\theta - \delta\cos\theta}, \tag{2.10}$$

where in the static limit of the quark model the parameter δ has the form [13]

$$|\delta| = \frac{m_s - m_u}{\sqrt{2}(m_s + m_u)} \approx 0.18\,. \tag{2.11}$$

The magnitude of $f_{K_1(1400)}/f_{K_1(1270)}$ can be determined from

$$\left(\frac{f_{K_1(1400)}}{f_{K_1(1270)}}\right)^2 = \frac{(m_\tau^2 + 2m_{K_1(1270)}^2)(m_\tau^2 - m_{K_1(1270)}^2)^2 m_{K_1(1400)}^2}{(m_\tau^2 + 2m_{K_1(1400)}^2)(m_\tau^2 - m_{K_1(1400)}^2)^2 m_{K_1(1270)}^2} \frac{\Gamma(\tau \to K_1(1400)\nu_\tau)}{\Gamma(\tau \to K_1(1270)\nu_\tau)}. \tag{2.12}$$

A fit of Eqs. (2.10) and (2.12) to the central value of the experimental measurement of R, the ratio of $K_1(1270)\nu_\tau$ to $K_1(1400)\nu_\tau$ [see Eq. (2.5)], yields

$$\begin{aligned} \theta &= \pm 37^\circ \quad \text{for } \delta = \mp 0.18\,, \\ \theta &= \pm 58^\circ \quad \text{for } \delta = \pm 0.18\,. \end{aligned} \tag{2.13}$$

Note that these solutions for the mixing angle are consistent with the ones $|\theta| \approx 33^\circ$ and 57° obtained in [13] based on the partial rates of K_1. However, contrary to the previous claim by Suzuki, $|\theta| \approx 58^\circ$ is still a possible solution allowed by $\tau \to K_1\nu_\tau$ decays. In the present work we will try to see if one of the remaining two solutions will be picked up by the study of $D \to K_1\pi$ decays.*

Although the data on $\tau \to a_1(1260)\nu_\tau \to \rho\pi\nu_\tau$ have been reported by various experiments (for a review, see [20]), the decay $\tau \to a_1(1260)\nu_\tau$ is not shown in the Particle Data Group [7]. Nevertheless, an experimental value of $f_{a_1} = 203 \pm 18$ MeV is quoted in [21]. It is generally argued that $a_1(1260)$ should have a similar decay constant as the ρ meson. This is confirmed by the model calculation, see e.g. [22]. For definiteness, we choose the $a_1(1260)$ decay constant to be 205 MeV.

B. Form factors

Form factors for the $D \to P$ transition are defined by [23]

$$\langle P(p)|V_\mu|D(p_D)\rangle = \left(p_{D\mu} + p_\mu - \frac{m_D^2 - m_P^2}{q^2}\, q_\mu\right) F_1^{DP}(q^2) + \frac{m_D^2 - m_P^2}{q^2} q_\mu\, F_0^{DP}(q^2), \tag{2.14}$$

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*As pointed out by Suzuki [19], the relation $|M(J/\psi \to K_1^0(1400)\overline{K}^0)|^2 = \tan^2\theta |M(J/\psi \to K_1^0(1270)\overline{K}^0)|^2$ will be able to determine θ directly without referring to other parameters. However, these decays have thus far not been measured.

where $q_\mu = (p_D - p)_\mu$. One of the form factors relevant for $D \to AP$ decays is $F_1^{DP}(q^2)$. To compute this form factor we will use the Bauer-Stech-Wirbel (BSW) model [23] which adopts the pole dominance assumption for the form-factor momentum dependence

$$f(q^2) = \frac{f(0)}{(1 - q^2/m_*^2)^n}, \tag{2.15}$$

with m_* being the 1^- (0^-) pole mass for F_1 (F_0). The original BSW model assumes a monopole behavior (i.e. $n = 1$) for all the form factors. However, this is not consistent with heavy quark symmetry scaling relations for heavy-to-light transitions. The modified BSW model takes the BSW model results for the form factors at zero momentum transfer but makes a different ansatz for their q^2 dependence, namely, a dipole behavior (i.e. $n = 2$) is assumed for the form factors F_1, V_0, V_2, A, motivated by heavy quark symmetry, and a monopole dependence for F_0, V_1, where the form factors V_i and A will be introduced shortly.

In the Isgur-Scora-Grinstein-Wise (ISGW) model [8,9], the vector form factors for $D \to A$ transition are defined by

$$\begin{aligned}
\langle A(p_A,\varepsilon)(^3P_1)|V_\mu|D(p_D)\rangle &= \ell\varepsilon^*_\mu + c_+(\varepsilon^*\cdot p_D)(p_D+p_A)_\mu + c_-(\varepsilon^*\cdot p_D)(p_D-p_A)_\mu, \\
\langle A(p_A,\varepsilon)(^1P_1)|V_\mu|D(p_D)\rangle &= r\varepsilon^*_\mu + s_+(\varepsilon^*\cdot p_D)(p_D+p_A)_\mu + s_-(\varepsilon^*\cdot p_D)(p_D-p_A)_\mu.
\end{aligned} \tag{2.16}$$

The form factors ℓ, c_+, c_-, r, s_+ and s_- can be calculated in the ISGW quark model [8] and its improved version, the ISGW2 model [9]. In general, the form factors evaluated in the ISGW model are reliable only at $q^2 = q_m^2 \equiv (m_D - m_A)^2$, the maximum momentum transfer. The reason is that the form-factor q^2 dependence in the ISGW model is proportional to $\exp[-(q_m^2 - q^2)]$ (see Eq. (A7)) and hence the form factor decreases exponentially as a function of $(q_m^2 - q^2)$. This has been improved in the ISGW2 model in which the form factor has a more realistic behavior at large $(q_m^2 - q^2)$ which is expressed in terms of a certain polynomial term (see Eq. (A1)). In addition to the form-factor momentum dependence, the ISGW2 model incorporates a number of improvements, such as the constraints imposed by heavy quark symmetry, hyperfine distortions of wave functions, etc.,$\cdots$ [9].

Note that the results for the form factor c_+ are quite different in the ISGW and ISGW2 models (see Table II): c_+ is positive in the former model while it becomes negative in the latter (see the Appendix for details).

In realistic calculations of decay amplitudes it is convenient to use the dimensionless form factors defined by [23]

$$\begin{aligned}
\langle A(p_A,\varepsilon)|V_\mu|D(p_D)\rangle &= \Bigg\{(m_D+m_A)\varepsilon^*_\mu V_1^{DA}(q^2) - \frac{\varepsilon^*\cdot p_D}{m_D+m_A}(p_D+p_A)_\mu V_2^{DA}(q^2) \\
&\quad - 2m_A\frac{\varepsilon^*\cdot p_D}{q^2}(p_D-p_A)_\mu\left[V_3^{DA}(q^2) - V_0^{DA}(q^2)\right]\Bigg\}, \\
\langle A(p_A,\varepsilon)|A_\mu|D(p_D)\rangle &= \frac{2}{m_D+m_A} i\epsilon_{\mu\nu\rho\sigma}\varepsilon^{*\nu}p_D^\rho p_A^\sigma A^{DA}(q^2),
\end{aligned} \tag{2.17}$$

with

TABLE II. The form factors at $q^2 = m_K^2$ for $D \to a_1$ and $D \to b_1$ transitions and at $q^2 = m_\pi^2$ for $D \to K_{1A}$ and $D \to K_{1B}$ transitions, where ℓ and r are in units of GeV and others carry units of GeV^{-1}. The first entry is for the form factors calculated in the ISGW model and the second entry is for the ISGW2 model.

Transition	ℓ	c_+	c_-	r	s_+	s_-
$D \to a_1$	-0.93	0.20				
	-1.31	-0.11	-0.037			
$D \to b_1$				0.95	0.42	
				1.29	0.20	-0.072
$D \to K_{1A}$	-0.49	0.12				
	-0.78	-0.13	-0.030			
$D \to K_{1B}$				0.64	0.31	
				0.94	0.21	-0.051

$$V_3(q^2) = \frac{m_D + m_A}{2m_A} V_1(q^2) - \frac{m_D - m_A}{2m_A} V_2(q^2), \tag{2.18}$$

and $V_3(0) = V_0(0)$. Note that only the form factor V_0 will contribute to the factorizable amplitude as one can check the matrix element $q^\mu \langle A(p_A, \varepsilon)|V_\mu|D(p_D)\rangle$. The ISGW and ISGW2 model predictions for the form factors $V_{0,1,2}$ are exhibited in Table III.

TABLE III. The dimensionless vector form factors $V_{0,1,2}$ at $q^2 = m_K^2$ for $D \to a_1$ and $D \to b_1$ transitions and at $q^2 = m_\pi^2$ for $D \to K_{1A}$ and $D \to K_{1B}$ transitions calculated in the ISGW2 model. The numbers in parentheses are the results obtained using the ISGW model.

Transition	V_0	V_1	V_2
$D \to a_1$	-0.63 (-0.22)	-0.42 (-0.30)	0.35 (-0.63)
$D \to b_1$	0.68 (0.72)	0.42 (0.31)	-0.62 (-1.29)
$D \to K_{1A}$	-0.37 (-0.11)	-0.24 (-0.15)	0.40 (-0.39)
$D \to K_{1B}$	0.50 (0.45)	0.29 (0.20)	-0.65 (-0.99)

III. $D \to AP$ DECAYS

We will study some of the Cabibbo-allowed $D \to AP$ decays (A: axial-vector meson, P: pseudoscalar meson) within the framework of generalized factorization in which the hadronic decay amplitude is expressed in terms of factorizable contributions multiplied by the *universal* (i.e. process independent) effective parameters a_i that are renormalization scale and scheme independent. More precisely, the weak Hamiltonian has the form

$$H_{\rm eff} = \frac{G_F}{\sqrt{2}} V_{cs} V_{ud}^* \Big[a_1 (\bar{u}d)(\bar{s}c) + a_2 (\bar{s}d)(\bar{u}c) \Big] + h.c., \tag{3.1}$$

with $(\bar{q}_1 q_2) \equiv \bar{q}_1 \gamma_\mu (1-\gamma_5) q_2$. For hadronic charm decays, we shall use $a_1 = 1.15$ and $a_2 = -0.55$. The parameters a_1 and a_2 are related to the Wilson coefficients via

$$a_1 = c_1(\mu) + c_2(\mu)\left(\frac{1}{N_c} + \chi_1(\mu)\right), \qquad a_2 = c_2(\mu) + c_1(\mu)\left(\frac{1}{N_c} + \chi_2(\mu)\right), \tag{3.2}$$

where the nonfactorizable terms $\chi_i(\mu)$ will compensate the scale and scheme dependence of Wilson coefficients $c_i(\mu)$ to render a_i physical.

In terms of the topological amplitudes: T, the color-allowed external W-emission tree diagram; C, the color-suppressed internal W-emission diagram; E, the W-exchange diagram, the Cabibbo-allowed $D \to A\pi$ ($A = K_1(1270),\ K_1(1400)$) and $D \to \overline{K}A$ ($A = a_1(1260),\ b_1(1235)$) amplitudes have the expressions:

$$\begin{aligned} A(D^0 \to A^- \pi^+) &= T + E, \qquad A(D^0 \to A^0 \pi^0) = \frac{1}{\sqrt{2}}(C' - E), \\ A(D^+ \to A^0 \pi^+) &= T + C', \end{aligned} \tag{3.3}$$

and

$$\begin{aligned} A(D^0 \to K^- A^+) &= T' + E, \qquad A(D^0 \to \overline{K}^0 A^0) = \frac{1}{\sqrt{2}}(C - E), \\ A(D^+ \to \overline{K}^0 A^+) &= T' + C. \end{aligned} \tag{3.4}$$

For $D \to AP$ and $D \to PA$ decays, one can have two different external W-emission and internal W-emission diagrams, depending on whether the emission particle is a scalar meson or a pseudoscalar one. We thus denote the prime amplitudes T' and C' for the case when the scalar meson is an emitted particle [24].

A. $D \to K a_1(1260)$ and $D \to K b_1(1235)$

Under the factorization approximation, the $D \to K a_1(1260)$ and $D \to K b_1(1235)$ decay amplitudes read (the overall $\varepsilon^* \cdot p_D$ terms being dropped for simplicity)

$$\begin{aligned} A(D^+ \to \overline{K}^0 a_1^+(1260)) &= \frac{G_F}{\sqrt{2}} V_{cs} V_{ud}^* \left[2a_1 f_{a_1} m_{a_1} F_1^{DK}(m_{a_1}^2) + 2a_2 f_K m_{a_1} V_0^{Da_1}(m_K^2) \right], \\ A(D^0 \to K^- a_1^+(1260)) &= \frac{G_F}{\sqrt{2}} V_{cs} V_{ud}^* \, 2a_1 f_{a_1} m_{a_1} F_1^{DK}(m_{a_1}^2), \\ A(D^0 \to \overline{K}^0 a_1^0(1260)) &= \frac{G_F}{2} V_{cs} V_{ud}^* \, 2a_2 f_K m_{a_1} V_0^{Da_1}(m_K^2), \end{aligned} \tag{3.5}$$

and

$$
\begin{aligned}
A(D^+ \to \overline{K}^0 b_1^+(1235)) &= \frac{G_F}{\sqrt{2}} V_{cs} V_{ud}^* \left[2a_1 f_{b_1} m_{b_1} F_1^{DK}(m_{b_1}^2) + 2a_2 f_K m_{b_1} V_0^{Db_1}(m_K^2)\right], \\
A(D^0 \to K^- b_1^+(1235)) &= \frac{G_F}{\sqrt{2}} V_{cs} V_{ud}^*\, 2a_1 f_{b_1} m_{b_1} F_1^{DK}(m_{b_1}^2), \\
A(D^0 \to \overline{K}^0 b_1^0(1235)) &= \frac{G_F}{2} V_{cs} V_{ud}^*\, 2a_2 f_K m_{b_1} V_0^{Db_1}(m_K^2),
\end{aligned} \tag{3.6}
$$

where the factorizable W-exchange amplitude has been neglected owing to helicity and color suppression.

As mentioned in the Introduction, the branching ratios of the decays $D^0 \to K^- a_1^+(1260)$ and $D^+ \to \overline{K}^0 a_1^+(1260)$ have been predicted to be of order 1.5% and 3.8%, respectively [1] which are well below the measured values of $(7.2 \pm 1.1)\%$ and $(8.1 \pm 1.7)\%$ (see Table IV). In our study, the $\overline{K}^0 a_1^+$ rate gets enhanced for two reasons: (i) The q^2 dependence of the form factor $F_1^{DK}(q^2)$ is of the dipole rather than the monopole form in order to be consistent with heavy quark symmetry.† (ii) Contrary to [1] where the form factor $V_0^{Da_1}$ is assumed to be zero, the calculated form factor using the ISGW2 model yields a negative V_0 for $D \to a_1$ transition and a positive one for $D \to b_1$. This means that the interference between external and internal W-emission amplitudes is constructive in $D^+ \to \overline{K}^0 a_1^+(1260)$ and destructive in $D^+ \to \overline{K}^0 b_1^+(1235)$. Our result for the former is slightly larger than experiment (see Table IV). Recall that this mode has been measured by two different groups with the branching ratios of $(11.6\pm3.7)\%$ by E691 [25] and $(7.5\pm1.6)\%$ by Mark III [26]. Therefore, our result is in good agreement with E691. In view of this, it is important to have a refined measurement of this decay mode.

As for $D^0 \to K^- a_1^+(1260)$, the dipole q^2 dependence of the form factor F_1^{DK} will enhance its branching ratio from 1.7% to 3.8% (see the second column of Table IV). However, it is still smaller than experiment by a factor of 2. This is ascribed to the fact that we have so far neglected the W-exchange contribution. It has been noticed that a large long-distance W-exchange can be induced from final-state rescattering (see e.g. [28]). The data analysis of Cabibbo-allowed $D \to \overline{K}\rho$ decays indicates [11]

$$
\frac{E}{T}\bigg|_{D\to\overline{K}\rho} \approx 0.54\, e^{-i72^\circ}, \qquad \frac{E}{C}\bigg|_{D\to\overline{K}\rho} \approx 1.12\, e^{i76^\circ}. \tag{3.7}
$$

If we assume that this result holds also for $D \to \overline{K}A$ ($A = a_1(1260),\ b_1(1235)$), then the branching ratio will be enhanced to 6.2% as shown on the third column of Table IV. We also

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†If we use the Melikhov-Stech (MS) model [27] to evaluate the $D \to K$ transition form factor, the branching ratios will become 6.9% and 3.3%, respectively, for $\overline{K}^0 a_1^+$ and $K^- a_1^+$. This implies that the increase of $F_1^{DK}(q^2)$ at $q^2 = m^2_{a_1(1260)}$ is not fast enough in this phenomenological model. More precisely, $F_1^{DK}(0) = 0.78$ and $F_1^{DK}(m_{a_1}^2) = 1.29$ in the MS model, while the corresponding values are 0.76 and 1.75 in the improved BSW model.

TABLE IV. Branching ratios for $D \to K a_1(1260)$ and $D \to K b_1(1235)$.

Decay	Theory		Experiment [7]
	without FSIs	with FSIs	
$D^+ \to \overline{K}^0 a_1^+(1260)$	12.1%	12.1%	$(8.1 \pm 1.7)\%$
$D^0 \to K^- a_1^+(1260)$	3.8%	6.2%	$(7.2 \pm 1.1)\%$
$D^0 \to \overline{K}^0 a_1^0(1260)$	3.3×10^{-4}	5.6×10^{-4}	$< 1.9\%$
$D^+ \to \overline{K}^0 b_1^+(1235)$	1.7×10^{-3}	1.7×10^{-3}	
$D^0 \to K^- b_1^+(1235)$	3.7×10^{-6}	5.9×10^{-6}	
$D^0 \to \overline{K}^0 b_1^0(1235)$	3.9×10^{-4}	6.7×10^{-4}	

see that the FSI induced W-exchange will increase the branching ratio of $D^0 \to \overline{K}^0 a_1^0(1260)$ from 3.3×10^{-4} to 5.6×10^{-4}.

It is interesting to notice that although the phase space for the final state $\overline{K} a_1(1260)$ is substantially suppressed relative to $\overline{K}\rho$, the large $D \to K$ transition form factor at $q^2 = m_{a_1}^2$ and the negative form factor V_0 for $D \to a_1$ transition render $\mathcal{B}(D^+ \to \overline{K}^0 a_1^+) \gtrsim \mathcal{B}(D^+ \to \overline{K}^0 \rho^+)$ and $\mathcal{B}(D^0 \to K^- a_1^+) \lesssim \mathcal{B}(D^0 \to K^- \rho^+)$. However, $\mathcal{B}(D^0 \to \overline{K}^0 a_1^0) < \mathcal{B}(D^0 \to \overline{K}^0 \rho^0)$.

Owing to the smallness of the b_1 decay constant, the decay rates of $\overline{K}^0 b_1^+$ and $K^- b_1^+$ are much smaller than their counterparts $\overline{K}^0 a_1^+$ and $K^- a_1^+$. Nevertheless, the neutral modes $\overline{K}^0 b_1^0$ and $\overline{K}^0 a_1^0$ are comparable.

B. $D \to K_1(1270)\pi$ and $D \to K_1(1400)\pi$

The factorizable amplitudes for $D \to K_1(1270)\pi$ and $D \to K_1(1400)\pi$ are (the overall $\varepsilon^* \cdot p_D$ terms being dropped for simplicity)[‡]

$$\begin{aligned}
A(D^+ \to \overline{K}_1^0(1270)\pi^+) &= \frac{G_F}{\sqrt{2}} V_{cs} V_{ud}^* \Big[2a_1 m_{K_1(1270)} f_\pi (\sin\theta\, V_0^{DK_{1A}}(m_\pi^2) + \cos\theta\, V_0^{DK_{1B}}(m_\pi^2)) \\
&\quad + 2a_2 m_{K_1(1270)} f_{K_1(1270)} F_1^{D\pi}(m_{K_1(1270)}^2) \Big], \\
A(D^+ \to \overline{K}_1^0(1400)\pi^+) &= \frac{G_F}{\sqrt{2}} V_{cs} V_{ud}^* \Big[2a_1 m_{K_1(1400)} f_\pi (\cos\theta\, V_0^{DK_{1A}}(m_\pi^2) - \sin\theta\, V_0^{DK_{1B}}(m_\pi^2)) \\
&\quad + 2a_2 m_{K_1(1400)} f_{K_1(1400)} F_1^{D\pi}(m_{K_1(1400)}^2) \Big], \\
A(D^0 \to K_1^-(1270)\pi^+) &= \frac{G_F}{\sqrt{2}} V_{cs} V_{ud}^* \Big[2a_1 m_{K_1(1270)} f_\pi (\sin\theta\, V_0^{DK_{1A}}(m_\pi^2) + \cos\theta\, V_0^{DK_{1B}}(m_\pi^2)) \Big],
\end{aligned}$$

[‡]In [5], the color-suppressed amplitudes in $D \to \overline{K}_1(1270)\pi$ and $\overline{K}_1(1400)\pi$ decays characterized by the parameter a_2 are erroneously multiplied by an additional factor of $\sin\theta$ and $\cos\theta$, respectively.

$$
\begin{aligned}
A(D^0 \to K_1^-(1400)\pi^+) &= \frac{G_F}{\sqrt{2}} V_{cs}V_{ud}^*\Big[2a_1 m_{K_1(1400)} f_\pi(\cos\theta\, V_0^{DK_{1A}}(m_\pi^2) - \sin\theta\, V_0^{DK_{1B}}(m_\pi^2))\Big], \\
A(D^0 \to \overline{K}_1^0(1270)\pi^0) &= \frac{G_F}{2} V_{cs}V_{ud}^*\Big[2a_2 m_{K_1(1270)} f_{K_1(1270)} F_1^{D\pi}(m_{K_1(1270)}^2)\Big], \\
A(D^0 \to \overline{K}_1^0(1400)\pi^0) &= \frac{G_F}{2} V_{cs}V_{ud}^*\Big[2a_2 m_{K_1(1400)} f_{K_1(1400)} F_1^{D\pi}(m_{K_1(1400)}^2)\Big],
\end{aligned} \tag{3.8}
$$

where we have taken into account the $K_{1A} - K_{1B}$ mixing given by Eq. (2.1). As before, we have neglected the short-distance factorizable W-exchange contribution.

Using the $D \to K_{1A}$ and $D \to K_{1B}$ form factors computed in the ISGW2 model (see Table III) and $f_{K_1(1270)} = 145$ MeV, the results for the branching ratios of $D \to K_1\pi$ are depicted in Table V for the mixing angles $|\theta| = 37^\circ$ and 58°. It is evident that the positive mixing-angle solutions $\theta = 37^\circ$ and 58° are ruled out as the predicted $\overline{K}_1^0(1270)\pi^+$ is too large while $K^-(1270)\pi^+$ is too small compared to experiment. Note that the experimental limit on $D^+ \to \overline{K}_1^0(1270)\pi^+$ is measured to be 0.007 by E691 [25] and 0.011 by Mark III [26]. Therefore, both negative mixing-angle solutions are allowed by experiment. However, $D^0 \to K_1^-(1400)\pi^+$ is very suppressed for $\theta \approx -37^\circ$. Hence an observation of this mode at the level of 5×10^{-4} will rule out $\theta \approx -37^\circ$ and favor the other solution $\theta \approx -58^\circ$.

TABLE V. Branching ratios of $D \to K_1(1270)\pi$ and $D \to K_1(1400)\pi$ calculated for various $K_{1A} - K_{1B}$ mixing angles.

Decay	Theory				Experiment [7]
	-37°	-58°	37°	58°	
$D^+ \to \overline{K}_1^0(1270)\pi^+$	6.4×10^{-3}	7.8×10^{-3}	2.9×10^{-2}	4.7×10^{-2}	$< 7 \times 10^{-3}$
$D^+ \to \overline{K}_1^0(1400)\pi^+$	2.9×10^{-2}	4.0×10^{-2}	6.6×10^{-2}	6.6×10^{-2}	$(4.9 \pm 1.2)\%$
$D^0 \to K_1^-(1270)\pi^+$	6.3×10^{-3}	5.5×10^{-3}	4.9×10^{-4}	4.4×10^{-5}	$(1.13 \pm 0.31)\%$
$D^0 \to K_1^-(1400)\pi^+$	3.7×10^{-8}	4.2×10^{-4}	3.0×10^{-3}	3.2×10^{-3}	$< 1.2\%$
$D^0 \to \overline{K}_1^0(1270)\pi^0$	8.4×10^{-3}	8.4×10^{-3}	8.4×10^{-3}	8.4×10^{-3}	$< 2.0\%$
$D^0 \to \overline{K}_1^0(1400)\pi^0$	5.7×10^{-3}	5.5×10^{-3}	5.7×10^{-3}	5.5×10^{-3}	$< 3.7\%$

Several remarks are in order. (i) For the decay constant of $K_1(1270)$, we use the value of 145 MeV rather than 175 MeV as inferred from the $\tau \to K_1(1270)\nu_\tau$ decay. If the latter is used, we will have $\mathcal{B}(D^+ \to \overline{K}_1^0(1270)\pi^+) = 1.5\%$ and 1.7%, respectively, for $\theta = -37^\circ$ and -58°, which exceed the current experimental limit. (ii) In Table V we have not taken into account the W-exchange contributions. If we assume that the W-exchange term relative to the amplitudes T and C is similar to that in $D \to \overline{K}^*\pi$ decays, namely [11],

$$
\frac{E}{T}\Big|_{D\to\overline{K}^*\pi} \approx 0.78\, e^{i96^\circ}, \qquad \frac{E}{C}\Big|_{D\to\overline{K}^*\pi} \approx 0.94\, e^{i248^\circ}, \tag{3.9}
$$

the branching ratios of $\overline{K}_1^0(1270)\pi^0$ and $\overline{K}_1^0(1400)\pi^0$ will become 2.2% and 1.4%, respectively. The former slightly exceeds the current limit. Therefore, the realistic value of W-exchange is smaller than that given by Eq. (3.9). (iii) We see that $\overline{K}_1^0(1400)\pi^+$ is larger than $\overline{K}_1^0(1270)\pi^+$ by one order of magnitude since the interference between color-allowed and color-suppressed amplitudes is constructive in the latter and destructive in the former. (iv) Though the decays $D^0 \to \overline{K}_1^0\pi^0$ are color suppressed, they are comparable to and even larger than the color-allowed counterparts: $\overline{K}_1^0(1270)\pi^0 \sim K_1^-(1270)\pi^+$ and $\overline{K}_1^0(1400)\pi^0 > K_1^-(1400)\pi^+$. This can be seen from Eq. (3.8) and from the fact that the form factor V_0 is negative (positive) for $D \to K_{1A}$ ($D \to K_{1B}$) transition and that $F_1^{D\pi}$ is large at $q^2 = m^2_{K_1(1270)}$ or $m^2_{K_1(1400)}$. Since the inclusion of the W-exchange contribution will enhance the decay rates of $\overline{K}_1^0(1270)\pi^0$ and $\overline{K}_1^0(1400)\pi^0$ by a factor of, say 1.5, it is conceivable that $D^0 \to \overline{K}_1^0\pi^0$ has a branching ratio of order 10^{-2}. Hence, the neutral $\overline{K}_1^0\pi^0$ modes should be easily accessible by experiment.

C. Finite width effect

Among the four axial-vector mesons we have studied thus far, $a_1(1260)$ is a broad resonance with a large width ranging from 250 MeV to 600 MeV and hence it will increase the phase space available. A running mass for the resonance has been considered in [1] to take into account the smearing effect due to the large width. However, the ansatz of a Breit-Wigner measure $\rho(m^2)$ made in [1] is somewhat arbitrary.

The factorization relation

$$\Gamma(D \to RM \to M_1M_2M) = \Gamma(D \to RM)\mathcal{B}(R \to M_1M_2), \tag{3.10}$$

which is often employed is, strictly speaking, valid only in the narrow width approximation. For an illustration, we consider the decay $D \to \overline{K}a_1(1260) \to \overline{K}\pi\pi\pi$. Following [31], we compute the quantity

$$\eta \equiv \frac{\Gamma(D \to \overline{K}a_1(1260) \to \overline{K}\rho\pi \to \overline{K}\pi\pi\pi)}{\Gamma(D \to \overline{K}a_1(1260))\mathcal{B}(a_1 \to \rho\pi \to \pi\pi\pi)}, \tag{3.11}$$

where we have assumed that $a_1(1260)$ decays entirely into $\rho\pi$ [7]. The deviation of η from unity will give a measure of the violation of the factorization relation. Owing to the finite width effect, the effective decay rate of $D \to \overline{K}a_1(1260)$ becomes

$$\Gamma(D \to \overline{K}a_1(1260))_{\rm fw} = \eta\,\Gamma(D \to \overline{K}a_1(1260)). \tag{3.12}$$

To proceed we write the on-shell decay amplitudes as

$$A(D \to \overline{K}a_1) = M(D \to \overline{K}a_1)(\varepsilon^* \cdot p_D), \qquad A(a_1 \to \rho\pi) = (gG_{\mu\nu} + hL_{\mu\nu})\varepsilon^\mu_{a_1}\varepsilon^{*\nu}_\rho, \tag{3.13}$$

where [21]

$$\begin{aligned} G^{\mu\nu} &= \delta^{\mu\nu} - \frac{1}{Y}\left[m^2_{a_1}p^\mu_\rho p^\nu_\rho + m^2_\rho p^\mu_{a_1}p^\nu_{a_1} + p_{a_1}\cdot p_\rho(p^\mu_{a_1}p^\nu_\rho + p^\mu_\rho p^\nu_{a_1})\right], \\ L^{\mu\nu} &= \frac{p_{a_1}\cdot p_\rho}{Y}\left(p^\mu_{a_1} + p^\mu_\rho\frac{m^2_{a_1}}{p_{a_1}\cdot p_\rho}\right)\left(p^\nu_\rho + p^\nu_{a_1}\frac{m^2_\rho}{p_{a_1}\cdot p_\rho}\right), \end{aligned} \tag{3.14}$$

and $Y = (p_{a_1} \cdot p_\rho)^2 - m_{a_1}^2 m_\rho^2$. The two-body decay rates then read

$$\begin{aligned} \Gamma(D \to \overline{K} a_1) &= \frac{p^3}{8\pi m_D^2} |M(D \to \overline{K} a_1)|^2, \\ \Gamma(a_1 \to \rho\pi) &= \frac{p'}{12\pi m_{a_1}^2} |M(a_1 \to \rho\pi)|^2, \end{aligned} \tag{3.15}$$

where [21]

$$|M(a_1 \to \rho\pi)|^2 = \left(2|g|^2 + \frac{m_{a_1}^2 m_\rho^2}{(p_{a_1} \cdot p_\rho)^2} |h|^2 \right), \tag{3.16}$$

p is the c.m. momentum of $\overline{K}$ or a_1 in the D rest frame, and p' is the c.m. momentum of the ρ or π in the a_1 resonance rest frame.

The resonant three-body decay rate is given by

$$\begin{aligned} \Gamma(D \to \overline{K} a_1 \to \overline{K} \rho\pi) = \frac{1}{8m_D^3} \int_{(m_\rho+m_\pi)^2}^{(m_D-m_K)^2} \frac{dq^2}{2\pi} |M(D \to \overline{K} a_1)|^2 \, |M(a_1 \to \rho\pi)|^2 \\ \times \frac{\lambda^{3/2}(m_D^2, q^2, m_K^2)}{8\pi m_D^2} \, \frac{\lambda^{1/2}(q^2, m_\rho^2, m_\pi^2)}{12\pi q^2} \, \frac{1}{(q^2 - m_{a_1}^2)^2 + (\Gamma_{\rho\pi}(q^2) m_{a_1})^2}, \end{aligned} \tag{3.17}$$

where λ is the usual triangluar function $\lambda(a,b,c) = a^2 + b^2 + c^2 - 2ab - 2ac - 2bc$, and the "running" or "comoving" width $\Gamma_{\rho\pi}(q^2)$ is a function of the invariant mass squared $m_{\rho\pi}^2 = q^2$ of the $\rho\pi$ system and it has the expression [29]

$$\Gamma_{\rho\pi}(q^2) = \Gamma_{a_1} \frac{m_{a_1}}{m_{\rho\pi}} \left(\frac{p'(q^2)}{p'(m_{a_1}^2)} \right)^3 \frac{1 + R^2 p'^2(m_{a_1}^2)}{1 + R^2 p'^2(q^2)}, \tag{3.18}$$

where $p'(q^2) = \lambda^{1/2}(q^2, m_\rho^2, m_\pi^2)/(2\sqrt{q^2})$ and we follow [30] to take R, the "radius" of the meson, to be $1.5\,\mathrm{GeV}^{-1}$. When the resonance width Γ_{a_1} is narrow, the expression of the resonant decay rate can be simplified by applying the so-called narrow width approximation

$$\frac{1}{(q^2 - m_{a_1}^2)^2 + m_{a_1}^2 \Gamma_{\rho\pi}^2(q^2)} \approx \frac{\pi}{m_{a_1} \Gamma_{a_1}} \, \delta(q^2 - m_{a_1}^2). \tag{3.19}$$

It is easily seen that this leads to the factorization relation Eq. (3.10) for the resonant three-body decay.

Assuming that $|M(D \to \overline{K} a_1)|^2$ and $|M(a_1 \to \rho\pi)|^2$ are insensitive to the q^2 dependence when the resonance is off its mass shell, these terms will be dropped in the expression of the parameter η. We find $\eta = 1.07$ and 1.22 for $\Gamma_{a_1(1260)} = 250$ MeV and 600 MeV, respectively. Note that our results disagree with [1] where the $a_1(1260)$ mass smearing procedure leads to *lower* the rate. The finite width effect becomes small for $b_1(1235)$, $K_1(1270)$ and $K_1(1400)$ production.

As stressed in [31], the finite width effect is most dramatic when the decay is marginally or even not allowed kinematically. For example, it is found that $\eta \sim 4.3$ for $D^0 \to f_0(1370)\overline{K}^0$

for $m_{f_0(1370)} = 1370$ MeV and $\Gamma_{f_0(1370)} = 500$ MeV. Evidently, the finite width effect of $f_0(1370)$ is very crucial for $D^0 \to f_0(1370)\overline{K}^0$. Recently, the branching ratios of $D^+ \to \overline{K}^{*0} a_1^+(1260)$ and $D_s^+ \to \phi a_1^+(1260)$ have been measured by FOCUS [32] based on the hypothesis that five-body modes are dominated by quasi-two-body decays. These modes are not kinematically allowed if $a_1(1260)$ is very narrow and on its mass shell. A study of these decays will appear in a forthcoming publication.

IV. CONCLUSIONS

Cabibbo-allowed charmed meson decays into a pseudoscalar meson and an axial-vector meson are studied. The charm to axial-vector meson transition form factors are evaluated in the Isgur-Scora-Grinstein-Wise quark model. The main conclusions are:

1. The $D \to A$ transition form factor c_+ has an opposite sign in the ISGW model and its improved version. It is found that the magnitude of the $D \to {}^3P_0$ form factor V_0 in the ISGW2 model is three times larger than that in the ISGW model.

2. The early predictions of $D^0 \to K^- a_1^+$ and $D^0 \to \overline{K}^0 a_1^+$ rates are too small by a factor of 5-6 and 2, respectively, when compared with experiment. The dipole momentum dependence of the form factor for the $D \to K$ transition, which is required by heavy quark symmetry, and the presence of a sizable long-distance W-exchange induced from final-state rescattering are the two key ingredients for understanding the data of $D \to \overline{K} a_1$. We predict that $\mathcal{B}(D^+ \to \overline{K}^0 a_1^+(1260)) = 12.1\%$, which is consist with E691 but slightly larger than the Mark III measurement. Experimentally, it is important to have a refined measurement of this decay mode.

3. $D \to \overline{K} b_1(1235)$ decays are in general suppressed relative to $D \to \overline{K} a_1(1260)$ owing to the smallness of the decay constant of b_1. However, the neutral modes $\overline{K}^0 b_1^0(1235)$ and $\overline{K}^0 a_1^0(1260)$ are comparable.

4. The $K_{1A} - K_{1B}$ mixing angle of the strange axial-vector mesons is extracted from $\tau \to K_1 \nu_\tau$ decays to be $\approx 37^\circ$ or 58° with a two-fold ambiguity. This is consistent with the mixing angles obtained from the experimental information on masses and the partial rates of $K_1(1270)$ and $K_1(1400)$. It is found that the positive mixing-angle solutions are excluded by the study of $D \to K_1(1270)\pi,\ K_1(1400)\pi$ decays. An observation of the decay $D^0 \to K_1^-(1400)\pi^+$ at the level of 5×10^{-4} will rule out $\theta \approx -37^\circ$ and favor the other solution $\theta \approx -58^\circ$.

5. Though the decays $D^0 \to \overline{K}_1^0 \pi^0$ are color suppressed, they are comparable to and even larger than the color-allowed counterparts: $\overline{K}_1^0(1270)\pi^0 \sim K_1^-(1270)\pi^+$ and $\overline{K}_1^0(1400)\pi^0 > K_1^-(1400)\pi^+$. It is expected that the neutral modes $D^0 \to \overline{K}_1^0 \pi^0$ have a branching ratio of order 10^{-2} and hence they should be easily accessible by experiment.

6. The finite width effect of the axial-vector resonance is studied It becomes important for $a_1(1260)$ especially when its width is near 600 MeV: The $D \to \overline{K} a_1(1260)$ rate is enhanced by a factor of 1.07 and 1.22, respectively, for $\Gamma_{a_1} = 200$ MeV and 600 MeV.

ACKNOWLEDGMENTS

This work was supported in part by the National Science Council of R.O.C. under Grant No. NSC91-2112-M-001-038.

APPENDIX

A. FORM FACTORS IN THE ISGW MODEL

Consider the transition $D \to A$, where the axial-vector meson A has the quark content $q_1\bar{q}_2$ with $\bar{q}_2$ being the spectator quark. We begin with the definition [9]

$$F_n = \left(\frac{\tilde{m}_A}{\tilde{m}_D}\right)^{1/2} \left(\frac{\beta_D\beta_A}{\beta_{DA}}\right)^n \left[1 + \frac{1}{18}h^2(t_m - t)\right]^{-3}, \tag{A1}$$

where

$$h^2 = \frac{3}{4m_c m_1} + \frac{3m_2^2}{2\bar{m}_D\bar{m}_A\beta_{DA}^2} + \frac{1}{\bar{m}_D\bar{m}_A}\left(\frac{16}{33-2n_f}\right)\ln\left[\frac{\alpha_s(\mu_{\rm QM})}{\alpha_s(m_1)}\right], \tag{A2}$$

$\tilde{m}$ is the sum of the meson's constituent quarks' masses, $\bar{m}$ is the hyperfine-averaged mass, $t_m = (m_D - m_A)^2$ is the maximum momentum transfer, and

$$\mu_\pm = \left(\frac{1}{m_1} \pm \frac{1}{m_c}\right)^{-1}, \tag{A3}$$

with m_1 and m_2 being the masses of the quarks q_1 and $\bar{q}_2$, respectively. In Eq. (A1), the values of the parameters β_D and β_A are available in [9] and $\beta_{DA}^2 = \frac{1}{2}(\beta_D^2 + \beta_A^2)$.

The form factors defined by Eq. (2.16) have the following expressions in the improved ISGW model:

$$\begin{aligned}
\ell &= -\tilde{m}_D\beta_D\left[\frac{1}{\mu_-} + \frac{m_2\tilde{m}_A(\tilde{\omega}-1)}{\beta_D^2}\left(\frac{5+\tilde{\omega}}{6m_1} - \frac{1}{2\mu_-}\frac{m_2}{\tilde{m}_A}\frac{\beta_D^2}{\beta_{DA}^2}\right)\right]F_5^{(\ell)},\\
c_+ + c_- &= -\frac{m_2\tilde{m}_A}{2m_1\tilde{m}_D\beta_D}\left(1 - \frac{m_1 m_2}{2\tilde{m}_A\mu_-}\frac{\beta_D^2}{\beta_{DA}^2}\right)F_5^{(c_++c_-)},\\
c_+ - c_- &= -\frac{m_2\tilde{m}_A}{2m_1\tilde{m}_D\beta_D}\left(\frac{\tilde{\omega}+2}{3} - \frac{m_1 m_2}{2\tilde{m}_A\mu_-}\frac{\beta_D^2}{\beta_{DA}^2}\right)F_5^{(c_+-c_-)},\\
r &= \frac{\tilde{m}_D\beta_D}{\sqrt{2}}\left[\frac{1}{\mu_+} + \frac{m_2\tilde{m}_A}{3m_1\beta_D^2}(\tilde{\omega}-1)^2\right]F_5^{(r)},\\
s_+ + s_- &= \frac{m_2}{\sqrt{2}\tilde{m}_D\beta_D}\left(1 - \frac{m_2}{m_1} + \frac{m_2}{2\mu_+}\frac{\beta_D^2}{\beta_{DA}^2}\right)F_5^{(s_++s_-)},\\
s_+ - s_- &= \frac{m_2}{\sqrt{2}\tilde{m}_D\beta_D}\left(\frac{4-\tilde{\omega}}{3} - \frac{m_1 m_2}{2\tilde{m}_A\mu_+}\frac{\beta_D^2}{\beta_{DA}^2}\right)F_5^{(s_+-s_-)},
\end{aligned} \tag{A4}$$

where

$$\begin{aligned}
F_5^{(\ell)} = F_5^{(r)} &= F_5\left(\frac{\bar{m}_D}{\tilde{m}_D}\right)^{1/2}\left(\frac{\bar{m}_A}{\tilde{m}_A}\right)^{1/2},\\
F_5^{(c_++c_-)} = F_5^{(s_++s_-)} &= F_5\left(\frac{\bar{m}_D}{\tilde{m}_D}\right)^{-3/2}\left(\frac{\bar{m}_A}{\tilde{m}_A}\right)^{1/2},\\
F_5^{(c_+-c_-)} = F_5^{(s_+-s_-)} &= F_5\left(\frac{\bar{m}_D}{\tilde{m}_D}\right)^{-1/2}\left(\frac{\bar{m}_A}{\tilde{m}_A}\right)^{-1/2},
\end{aligned} \tag{A5}$$

and

$$\tilde{\omega} - 1 = \frac{t_m - t}{2\bar{m}_D \bar{m}_A}. \tag{A6}$$

In the original version of the ISGW model [8], the function F_n has a different expression in its $(t_m - t)$ dependence:

$$F_n = \left(\frac{\tilde{m}_A}{\tilde{m}_D}\right)^{1/2} \left(\frac{\beta_D \beta_A}{\beta_{DA}}\right)^n \exp\left\{-\frac{m_2}{4\tilde{m}_D \tilde{m}_A} \frac{t_m - t}{\kappa^2 \beta_{DA}^2}\right\}, \tag{A7}$$

where $\kappa = 0.7$ is the relativistic correction factor. The form factors are then given by

$$\begin{aligned}
\ell &= -\tilde{m}_D \beta_D \left[\frac{1}{\mu_-} + \frac{m_2}{2\tilde{m}_D} \frac{t_m - t}{\kappa^2 \beta_D^2} \left(\frac{1}{m_1} - \frac{1}{2\mu_-} \frac{m_2}{\tilde{m}_A} \frac{\beta_D^2}{\beta_{DA}^2}\right)\right] F_5, \\
c_+ &= \frac{m_2 m_c}{4\tilde{m}_D \beta_D \mu_-} \left(1 - \frac{m_1 m_2}{2\tilde{m}_A \mu_-} \frac{\beta_D^2}{\beta_{DA}^2}\right) F_5, \\
s_+ &= \frac{m_2}{\sqrt{2}\tilde{m}_D \beta_D} \left(1 + \frac{m_c}{2\mu_-} - \frac{m_1 m_2 m_c}{4\mu_+ \mu_- \tilde{m}_A} \frac{\beta_D^2}{\beta_{DA}^2}\right) F_5.
\end{aligned} \tag{A8}$$

It is clear that the form factor c_+ has an opposite sign in the ISGW and ISGW2 models. Note that the expressions in Eq. (A4) in the ISGW2 model allow one to determine the form factors c_- and s_-, which vanish in the ISGW model.

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